A Novel Transceiver Architecture for Making Polar Codes Work Over Multipath Fading Channels as They Do Over AWGN Channels

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Abstract

Polar codes, recently accepted for adoption in 5G standard due to their excellent performance at a very low complexity compared to other competitive schemes in the literature, are deemed to be a strong candidate for the Internet of Things (IoT) applications as well due to meeting their requirements. However, since polar codes construction is naturally channel-dependent, there has recently been an increasing interest in addressing the challenge of making polar codes work in realistic fading environments as they do in a binary symmetric channel (BSC). Recent studies on polar codes for fading channels have mainly focused on constructing new specific polar codes suitable to particular fading channels. This results in a non-universal code structure, leading to continuous changes in the code structure based on the channel, which is not desirable in practice. To address this problem, we develop and propose a novel transceiver architecture which enables using the polar coding design of a binary input additive white Gaussian noise (BI-AWGN) channel for multi-path fading channels without causing any changes in the structure of the encoder and decoder sides. This is made possible by eliminating the channel fading effect so that a net AWGN channel can be seen at the input of a simple successive cancellation decoder (SCD). The novelty of the proposed solution lies in using a channel-based orthonormal transformation with optimal power allocation at the transmitter and another transformation at the receiver to make the net, effective channel seen by the SCD very similar to an AWGN channel. The obtained results show that the proposed design makes the bit error rate (BER) performance of polar codes over a frequency selective fading channel the same as that over an AWGN channel. As a plus, great advantage, the proposed design is found to be capable of providing confidentiality against eavesdropping receivers (i.e., unintended users) at the physical layer. Particularly,

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wireless physical layer security is delivered by the proposed scheme because of the fact that the transmitted data blocks are designed based on the channel of the legitimate receiver, which is naturally different from that of an eavesdropper, who is normally located several wavelength away from the legitimate receiver, and thus experiencing a different channel.

Index Terms

Polar codes, AWGN, multi-path channel, wireless communication, frequency selective channels, transceiver design, smart devices, IoT, 5G systems.

I. INTRODUCTION

Thanks to the rapid advancements and great developments in both computing and communication technologies, smart cities are becoming not only a reality but also more popular and spreading day by day. In fact, the deployment of smart, cognitive cities is expected to be very dominant in many parts of the world in the near future due to their many advantages and merits represented by making life easier, faster, simpler, and safer [1]–[4].

Smart cities are composed of massive amount of smart, intelligent devices that have sensing, computing, actuating and communication capabilities, designed in such a way that reduces human intervention through cognition and automation. These smart devices, commonly termed as Internet of Thing (IoT) devices, are expected to dominate smart cities infrastructure. Among the many design requirements of IoT devices, especially for ultra-reliability and low-latency communication (URLLC)-based 5G services, low complexity with high reliability communication comes as a first key priority besides energy efficiency, latency and security [5]. To meet this design goal, polar coding is proposed and adopted to be used in future 5G and beyond systems as a strong, attractive, and indispensable channel coding scheme [6], [7].

Polar codes, recently proposed by Arikan in [8], have driven enormous efforts by the wireless research community because of their provably capacity-achieving property alongside its low complexity. Specifically, polar codes can achieve the capacity of the binary symmetric channel (BSC), which is equivalent to an Additive White Gaussian Noise (AWGN) channel with binary inputs generated by binary phase shift keying (BPSK) modulator. However, polar codes are not universal, which means different polar codes are constructed and generated according to the specified value of signal-to-noise ratio (SNR), defined as the design-SNR. This issue becomes
even more problematic when the channel is fading due to having multi-path (which is the case in most wireless channels) as it requires new ways and methods to construct new channel-specific polar codes.

In an attempt to make polar codes work in fading channels as it does in BSC, several works and research studies have been performed. In [9], a hierarchical, fading channels-tailored polar coding scheme that uses nested coding is presented. It was shown that the scheme is capacity achieving and it can be extended to fading channels with multiple, but finite number of states.

In [10], polar codes are applied to wireless channels. A procedure for obtaining the Bhattacharyya parameters associated with AWGN and Rayleigh channels is presented. In [11], tracking lower and upper bounds on Bhattacharyya parameters of the bit subchannels was performed to construct good polar codes. In [12], the author analyzed polar coding strategies for the radio frequency (RF) channel with known channel state information (CSI) at both ends of the link and with known channel distribution information (CDI). Moreover, the investigation of polar coding for block-fading channel was performed in [13]. In [14], a polar coding scheme for fading channels was proposed. In particular, by observing the polarization of different binary symmetric channels over different fading blocks, each channel use, corresponding to a different polarization, is modeled as a binary erasure channel such that polar codes could be adopted to encode over blocks.

The authors in [15] analyzed the general form of the extrinsic information transfer curve of polar codes viewed as multilevel codes with multistage decoding. Based on this analysis, they proposed a graphical design methodology to construct polar codes for inter-symbol interference channels. In [16], a simple method for the construction of polar codes for Rayleigh fading channel was presented. The sub-channels induced by the polarizing transformation are modeled as multipath fading channels, and their diversity order and noise variance are tracked.

The polar codes are design exclusively for block fading channels in [17]. Specifically, the authors in [17] constructed polar codes tailored for block fading channels by treating fading as a kind of polarization and then matching it with code polarization. The obtained codes are demonstrated to deliver considerable gain compared to conventional polar BICM schemes. In [18], the same authors provided an alternative design of polar codes for block fading channels by treating the combination of modulation, fading, and coding as a single entity. This design is based on the fact that the bit channels are polarized not only by code, but also by fading and
modulation. This observation enables constructing polar codes by mapping code polarization with modulation and fading polarization. The obtained codes adapt to the fluctuation of the channel.

In [19], the authors studied the problem of polar coding for non-coherent block fading channels without considering any instantaneous channel state information. The scheme proposed by authors achieves the capacity of binary-input block fading channels with only channel distribution information. In [20], the authors proposed a polar coding scheme for orthogonal frequency division multiplexing (OFDM) systems under multi-path frequency selective fading channels. In this scheme [20], the codeword bits are permuted in such a way that the bits corresponding to the frozen bits are assigned to subcarriers causing frequent bit errors. This permutation can be considered as a kind of interleaving that can noticeably improve the polarization of the channel, thereby enhancing the bit error rate performance.

As seen from the literature, the previous studies on polar codes for fading channel have mainly focused on constructing new specific polar codes suitable to a particular fading channel. However, changing the polar codes construction based on the channel is not desirable in practice, since it would result in a continuous change and modification in the code construction based on the channel type, which is considered to be a cumbersome, complex and inefficient process especially for IoT-based applications.

Unlike previous works, where new specific codes are constructed based on the channel, in this work, we propose a generic design solution, which enables us to use the same polar coding design, adopted in AWGN channel, for fading channels. The design neither causes any change in the encoder and decoder sides nor degrades the reliability performance. This is made possible through canceling the channel fading effect by using special channel-based transformations along with optimal power allocation\(^1\), so that a net, effective AWGN channel can be seen at the input of the successive cancellation decoder (SCD), whose simplicity and low complexity make it attractive to polar codes. In this work, a frequency selective fading channel is considered, which is the most common observed channel in broadband wireless systems.

\(^1\)It should be stated that one obvious, common way to theoretically cancel the channel fading effect completely is to use channel inversion by means of applying zero forcing method at the transmitter [21]; however, this way is unfortunately impractical as it causes noise enhancement and a huge increase in the transmit power where it can go to infinity when the channel is in a deep fading situation.
Besides the aforementioned advantages, the proposed design is found to be capable of delivering confidentiality as a security service against eavesdropping receivers (i.e., unintended users) at the physical layer. Particularly, wireless physical layer security is delivered by the proposed scheme because of the fact that the transmitted data blocks are designed based on the channel of the legitimate receiver, which is naturally different from that of an eavesdropper, who is normally located several wavelength away from the legitimate receiver, and thus experiencing a different channel.

The rest of the paper is organized as follows. System model is described in Section II. The details of the developed design is revealed in Section III. Then, Simulation results are discussed in Section IV. Finally, conclusion and future works are drawn in Section VI.

Notations: Vectors are denoted by bold-small letters, whereas matrices are denoted by bold-large letters. $\mathbf{I}$ is the $N \times N$ identity matrix. The convolution operator is indicated by $(\ast)$. The transpose and conjugate transpose are symbolized by $(\cdot)^T$ and $(\cdot)^H$, respectively.
II. SYSTEM MODEL AND PRELIMINARIES

A single-input single-output (SISO) system, in which a transmitter (Tx) communicates with a receiver (Rx) as shown in Fig. 1, is assumed. All received signals exhibit multi-path slowly varying Rayleigh fading channels. The channel reciprocity property is adopted, where the downlink channel can be estimated from the uplink one, in a time division duplex (TDD) or hybrid systems (TDD with FDD) using channel sounding [22]. Thus, the channel is assumed to be known at both the transmitter and receiver sides [21], [23]. As per Fig. 1, a block of data bits denoted by \( \mathbf{u} \) is encoded using a binary polar coding scheme constructed at a fixed design-SNR as depicted in Fig. 2, where \( N \) is the code length, \( K \) is the length of the message sequence, and \( J \subseteq N \), \( |J| = K \) is the set of active indices known as information bit indices, corresponding to the good channel observations [8], [24]. The remaining \( N - K \) indices are called as frozen (inactive) bit indices. Here, \( N \) is a power of 2 and we define \( n = \log_2(N) \). For a \((N, K, J)\) polar code, the generator matrix \( \mathbf{G} \) is the \( n \)-fold Kronecker product of \( n \) copies of \( \mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \). Therefore, given a message vector \( \mathbf{u} \) of \( K \) information bits, a codeword \( \mathbf{x} \) is generated as follows:

\[
\mathbf{x} = \mathbf{u} \cdot \mathbf{G} = \mathbf{d} \cdot \mathbf{F}^{\otimes n} ,
\]

where \( \mathbf{d} \) is a vector of \( N \) bits including information bits such that \( \mathbf{d}_J = \mathbf{u} \), and \( \mathbf{d}_{J^c} = 0 \). The

![Fig. 2. Polar encoder structure with \((N, K, J) = (8, 5, \{2, 4, 6, 7, 8\})\).](image)
bits $d_x$ are called as frozen bits, and are set to zero.

The codeword $x$ is modulated using BPSK, resulting in a block of real modulated symbols, represented by

$$ s = \begin{bmatrix} s_0 & s_1 & \ldots & s_{N-1} \end{bmatrix}^T \in \mathbb{C}^{[N \times 1]}.$$  \hspace{1cm} (2)

In the proposed transmission scheme, each one of the real base-band modulated symbols $s_i$ is assigned a specific power value $e_i$ based on the quality of the corresponding experienced channel. For the $N$ data symbols to be transmitted without interference over a time dispersive channel, we need $N$ carrying orthogonal basis vectors, which can be taken in the proposed scheme from the column vectors of transformation matrix $V$, (whose design details will be explained in the next coming section) given by

$$ V = \begin{bmatrix} v_0 & v_1 & \ldots & v_{N-1} \end{bmatrix} \in \mathbb{C}^{[N \times N]}.$$  \hspace{1cm} (3)

Hence, $V$ can be seen as the channel-based transformation matrix, which changes based on the user’s channel. Also, each $i$th column vector in $V$ can be expressed as

$$ v_i = \begin{bmatrix} v_0 & v_1 & \ldots & v_{N-1} \end{bmatrix}^T \in \mathbb{C}^{[N \times 1]}.$$  \hspace{1cm} (4)

After multiplying each symbol with its corresponding basis vector, we take the summation (superposition) of all the resulting weighted vectors to get a block of samples $g$, referred to as one orthogonal transform division multiplexing (OTDM) symbol [25], [26]. The power allocation and transformation processes can mathematically be formulated as

$$ g = \sum_{i=0}^{N-1} e_i \cdot s_i \cdot v_i \in \mathbb{C}^{[N \times 1]},$$  \hspace{1cm} (5)

which can further be simplified into a linear matrix representation form as follows:

$$ g = V E^{-1} s \in \mathbb{C}^{[N \times 1]},$$  \hspace{1cm} (6)

where $E = diag \begin{bmatrix} e_0 & e_1 & \ldots & e_{N-1} \end{bmatrix}$ is a diagonal matrix, whose values represent the amount of power that is caused by each channel realization after transformation. Moreover, to avoid the interference between consecutive adjacent blocks, known as inter block interference (IBI), zero-suffix padding, as a guard period interval with length equals to the length of the channel delay spread $L$, is appended to the end of each block. Zero-padding in our design can be understood as an off-transmission period, as is the case in ZP-OFDM. This results in saving power.
resources compared to CP-OFDM, since no energy is sent in the guard period. Additionally, extra unnecessary extension in guard period is avoided since the guard period length is set to be equal to the channel spread. After that, the OTDM symbol is sent through $L$-path slowly varying frequency selective fading channel with impulse response given as

$$h = \begin{bmatrix} h_0 & h_1 & \ldots & h_{L-1} \end{bmatrix}^T,$$

(7)

whose elements $h_i$ are drawn from a complex Gaussian distribution function with zero mean and unity variance. The baseband received signal at the receiver can be given as

$$y = h \ast g + z$$

(8)

$$y_i = \sum_{i=0}^{L-1} h_l g(i-l) + z(i),$$

(9)

where $y = \begin{bmatrix} y_0 & y_1 & \ldots & y_{N+L-1} \end{bmatrix}^T$ is the received block of one OTDM symbol and $z \in \mathbb{C}^{(N+L-1) \times 1}$ is the zero-mean real additive white Gaussian noise (AWGN). The previous convolution form can also be equivalently written in a linear algebraic matrix form, as

$$y = Hg + z = HVE^{-1}s + z,$$

(10)

where $H \in \mathbb{C}^{(N+L-1) \times N}$ is the Toeplitz matrix form of the fading channel realizations between the transmitter and the receiver, given by

$$H = \begin{bmatrix} h_0 & 0 & 0 & \ldots & 0 \\ h_1 & h_0 & 0 & \ldots & 0 \\ h_2 & h_1 & h_0 & \ldots & 0 \\ \vdots & \vdots & h_1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{L-1} & h_{L-2} & \ldots & \ldots & \ldots \\ 0 & h_{L-1} & h_{L-2} & \ldots & \ldots \\ 0 & 0 & h_{L-1} & \ldots & \ldots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & h_{L-1} \end{bmatrix}.$$
At the receiver, a channel-based transformation is performed on \( y \), using a transformation matrix denoted by \( U \) that consists of multiple orthogonal basis vectors, which are optimally extracted from the channel to diagonalize the channel response. This process is then followed by successive cancellation decoding [8], [24] in the transform domain to decode the received data bits successfully. The process of extracting and using the matrix \( U \) will be discussed in the coming section.

III. PROPOSED TRANSCEIVER DESIGN

The main goal of the proposed design is to compensate the effect of multi-path frequency selective channel on the performance of polar codes, so that the required soft output symbols at the input of the successive cancellation decoder can be obtained without changing the polar code construction. This results in canceling the effect of fading and spreading caused by the multi-path channel, leading to a very good performance (close to that of AWGN channel).

The main design steps are explained as follows:

- Assuming that \( H \) is available at both communication sides (transmitter and receiver) by means of channel sounding before the communication starts, and since the channel impulse response can be modeled as a Toeplitz matrix for block-based transmission, then both the transmitter and receiver can decompose \( H \) by applying any of the common decomposition methods (such as singular value decomposition (SVD), uniform channel decomposition (UCD), and geometric mean decomposition (GMD), etc.). For familiarity and simplicity, we choose SVD as the underlying method, thus \( H \) can equivalently be expressed as follows:

\[
H = UEV^H. \quad (12)
\]

- The transmitter takes the inverse of the diagonal matrix \( E \), whose elements (singular values) represent the power spectrum of the parallel decomposed subchannels, and then uses it as a block-based power allocator (channel gain inverter or pre-equalizer).

- Before allocating the power to the modulated symbols, the transmitter classifies the \( N \) number of channel realizations into two group categories:
  1) Invertible group, which can be used for symbol transmission as it corresponds to fading channel realizations that can be compensated by optimal power allocation without causing
any increase in the transmit power (i.e., the average total transmit power remains unity).
2) Non-invertible group, which can not be used for symbol transmission as it requires a huge amount of transmit power to compensate for the effect of deep fading channel realizations. The determination of which elements are invertible and which are not is done by the transmitter via calculating the sum of the diagonal elements of matrix $E$ as follows

$$P = \sum_{i=0}^{N-1} e_i = e_0 + e_1 + e_2 + \ldots + e_{N-1}. \quad (13)$$

Based the value of $P$, the transmitter decides to select the first $\lfloor P \rfloor$ number of the channel realizations after transformation as carriers used for symbol transmission, whereas the remaining ones $(N - \lfloor P \rfloor)$ are suppressed (deactivated) and not used for symbol transmission. It should be noticed that this selection mechanism can result in a slight degradation in the effective throughput of the system due to the fact that the bad channel realizations, which are responsible for degrading the reliability performance of the system, are not used for data transmission, but rather suppressed and excluded from being used for symbol transmission.

- The transmitter and receiver take the hermitian (conjugate transpose) of the right and left matrices, resulting from applying SVD decomposition on $H$, i.e., $V^H$ and $U$, to get $V$ and $U^H$, respectively. Then, use the matrices as transformer and de-transformer at the transmitter and receiver sides, respectively, to diagonalize the channel response and make it resilient to inter symbol interference.

It should be noted that the transform domain (analogous to the frequency domain) is obtained due to using $V$ as an inverse fast Fourier transform (IFFT) at the transmitter and $U^H$ as a fast Fourier transform (FFT) at the receiver. Without loss of generality, we first use the columns of $V$ as orthogonal waveforms to carry the data symbols. The process of allocating channel gain-based power and assigning data symbols to waveforms and then summing them all can easily be expressed in a matrix form as in (6). When $g$ passes through the channel and reaches the receiver side, the received OTDM block becomes as follows:

$$y = HVE^{-1}s + z \in \mathbb{C}^{(N+L-1) \times 1}. \quad (14)$$

As seen from the previous equation, since $H$ can equivalently be written in terms of its SVD decomposition, then the pre-transformation matrix $V$ used at the transmitter cancels
the effect of the right part $V^H$ of the channel since their multiplication results in an identity matrix ($I$). Thus, the net received signal can be reformulated as

$$y = Us + z \in \mathbb{C}^{(N+L-1) \times 1}. \quad (15)$$

- To remove the effect of the time dispersion brought by the channel spread caused by the left part of the channel $U \in \mathbb{C}^{(N+L-1) \times N}$, the receiver needs to multiply the received signal by $U^H$ as follows:

$$U^H y = s + U^H z = s + \hat{z} \in \mathbb{C}^{N \times 1}. \quad (16)$$

where $\hat{z} = U^H z$, and because of the unitary nature of matrix $U^H$, $\hat{z}$ has the same statistics as $z$.

It should also be noted that the vectors of $U$ span not only the whole transmitted block time but also the following time reserved for zero-padding. Thus, $U$ takes into account the spreading caused by the channel. The column vectors of $U$ not only separate efficiently the transmitted symbols, leading to no interference between them, but also provide the best compromise between the useful energy and the noise collected in the zero-padding interval, leading to a maximum SNR increase for each transmitted block.

After multiplying by $U^H$, the leakage energy of the signal due to channel spreading, will be collected from the guard band optimally with minimal noise thanks to the adaptive orthogonal waveforms, whose length at the receiver is equal to the received block length. Specifically, $U^H$ maps the received block $y$ from $\mathbb{C}^{(N+L-1) \times 1}$ to $\mathbb{C}^{(N) \times 1}$, resulting in accumulating the leaked energy and automatically removing the inserted guard period (ZP). Now, since the multi-path channel is virtually converted to an equivalent AWGN channel as a result of applying the proposed signal processing scheme, a simple polar code decoder such as SCD can now be applied directly on the estimated block ($U^H y$).

It should be emphasized that since the transmitter performs channel gain inversion, the receiver no longer requires to perform equalization by $E$, which otherwise would cause significant noise enhancement, data distribution changes, etc. and thus limit the achievable performance.

Moreover, the proposed design provides not only reliability close to that of AWGN channel without changing the simple polar coding structure, but also physical layer security for providing
confidentiality against eavesdropping [23], [27]–[30] as an additional super advantage. Wireless physical security is delivered by the proposed scheme because of the fact that the transmitted data blocks are designed based on the channel of the legitimate receiver, which is naturally different than that of an eavesdropper’s one, who is normally located several wavelength away from the legitimate receiver, and thus experiencing a different channel.

IV. SIMULATION RESULTS

We consider a block-based polar coding scheme with length $N = \{64, 128, 256\}$ and code rate $R = 0.5$. The polar codes are constructed at a fixed design-SNR equals to zero using Arikan’s Bhattacharyya bounds [8] that is also explained in detail in [24]. BPSK modulation is then used, followed by optimal power allocation and orthonormal transformation as explained before in the previous section, and finally a guard period of length $L$ is appended to the transmitted block. At the receiver, a de-transformation process is performed, followed directly by a simple and low complexity decoder, named as SCD [8], [24].

In the simulator, the channel is modeled as an independent and identically distributed (i.i.d.) block-fading, where channel coefficients are drawn from an i.i.d. Rayleigh fading distribution, at the beginning of each block transmission, and remain constant within one block, but change independently from one to another. The Rayleigh multi-path fading channel has nine taps $L = 9$ with an exponential power delay profile given by

$$P_{pdp} = [0.8407, 0, 0, 0.1332, 0, 0.0168, 0.0067, 0, 0.0027] \text{ mW},$$

where the distribution of the amplitude of each non-zero tap is assumed to be Rayleigh as commonly used in the literature.

Fig. 3 presents the bit error rate (BER) performance versus $E_b/N_0$ (where $E_b$ is the bit energy and $N_0$ is the power spectral density of the noise) of polar codes using the proposed transceiver architecture. On the other hand, Fig. 4 presents the frame error rate (FER) performance versus $E_b/N_0$ of the proposed design at different block lengths.

It is shown from the both figures that the achieved performance over a multi-path fading channel is almost as same as that obtained over an AWGN channel [24]. The gain is attained as a result of canceling the dispersion and fading effects of the channel by performing proper transformation and optimal power allocation alongside channel selection according to the channel.
behavior in such a way that the total response of the system is diagonalized and becomes similar to that of an AWGN channel. This means that the inter symbol interference between data symbols $s_i$ within one transmission block, is totally removed.

Moreover, the way the interference leakage in the guard period is collected in the presence of noise, using $U^H$, is optimal. Additionally, it is clear from the BER performance that as the block length increases, the performance gets improved, which is exactly similar to what happens in the case of an AWGN channel [24].

It should be stated that the achieved desirable reliability performance (in terms of BER/FER) of the proposed scheme that makes polar codes designed for AWGN [24] work over multipath fading dispersive channels with the same reliability as that of AWGN comes at a cost. This cost is related to a slight random degradation (loss) in the maximum data rate that can be transmitted over the multipath channel as explained in the aforementioned section. Therefore, it would be
Fig. 4. BER performance of the proposed design over multi-path channel.

insightful to further understand and quantify the amount of data rate performance loss incurred by the proposed practical design.

To clearly show and visualize this loss, we conduct a simulation experiment by which we generate 10000 random multipath channel realizations whose power delay profile values are given as in (17). For each channel realization, whose length $N$ is assumed to be equal to 128 samples, we decompose the Toeplitz matrix form of the multipath channel using SVD method to obtain the diagonal matrix $E$ as explained in the aforementioned section. Once we have $E$, we can then calculate $P$, which is the sum of the diagonal elements of $E$. Since for each iteration we have one value for $P$, then we would have 10000 values for $P$ given the number of realizations we have which are 10000 as well. For the obtained 10000 values of $P$, we plot the probability density function (PDF) and cumulative distribution function (CDF) of $P$ as shown in Fig. 5 and Fig. 6, respectively. It can be noticed from the figures that the PDF and CDF
curves of the experimental values of $P$ fit to Nakagami distribution. Also, it is shown that the whole distribution region can be divided into two regions: 1) data rate lossless region, whose sum power is greater than the number of channel samples (i.e., $P > N$); and 2) data rate lossy region, whose sum power is less than the number of subcarriers, which are in this case equal to frequency subchannel samples, (i.e., $P < N$).

Accordingly, in the first region, we have fully invertible subchannels, whereas in the second region, we have partially invertible subchannels. Therefore, we would have a certain data rate loss in the lossy region as the the value of $P$ is less than $N$ and thus we cannot send data over all subchannels.

Furthermore, from Fig. 6, we can observe that the probability of being in the lossy region
is around 75%, which is quite high percentage, resulting in the need to activate only a certain number of subchannels according to the value of $P$. The number of deactivated subchannels (NDC) can be calculated as $NDC = N - \lfloor P \rfloor$, which increases as we move towards the left of the CDF curve, but with lower probability. This would result in limiting the amount of maximum data rate at which we can have a performance equal to that of an AWGN channel without changing the polar code design whatsoever. However, it should be pointed out that the deactivated subchannels can be utilized to do some other useful functionalities such as minimizing interference, mitigating out-of-band emission (OOBE), reducing peak-to-average power ratio (PAPR), and enhancing physical layer secrecy. [22].
In Fig. 7, the probability density function of the lossless region, whose $P$ values (i.e., sum of the diagonal elements of $E$) is greater than $N$, is plotted. As can be seen from the figure, the distribution can be fitted to a generalized extreme value distribution, which is much more closer to the empirical samples than log-normal distribution.

Besides, in Fig. 8, the probability density function of the lossy region, whose $P$ values (i.e., sum of the diagonal elements of $E$) is less than $N$, is plotted. As can be seen from the figure, the distribution can be fitted to a rician, weibull, or generalized extreme value distributions with different parameters.
Fig. 8. Probability distribution function (PDF) of lossy region $P < N$. 

$P$: Sum of the diagonal elements of $E$
V. CONCLUSION

In this work, we have proposed a novel solution that enables using the same polar coding design, used in AWGN channel, for multi-path frequency selective fading channels without causing any change in the encoder and decoder sides while maintaining the same reliability performance. This has been made possible by canceling the channel fading effect by using a pre-transformation with optimal power allocation at the transmitter and a post-transformation at the receiver, so that a net AWGN channel could be seen at the input of the successive cancellation decoder. Future work will include the extension of the proposed design to other practical modulations such as M-QAM, M-PSK, etc. Moreover, to limit the level of the transmit power and make the scheme compatible with practical power amplifiers, new methods are required to be designed. These future methods will be dedicated to utilizing the symbol subcarriers corresponding to low subchannel gains (i.e., the ones which are deactivated and not used for data transmission) for useful purposes and functionalities such as shaping the spectrum or reducing the peak to average power ratio (PAPR) of the transmit waveform.
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Author Biography

Jehad M. Hamamreh received the B.Sc. degree in electrical and telecommunication engineering from An-Najah University, Nablus, in 2013, and the Ph.D. degree in electrical electronics engineering and cyber systems from Istanbul Medipol University, Turkey, in 2018. He was a Researcher with the Department of Electrical and Computer Engineering, Texas A and M University at Qatar. He is currently an Assistant Professor with the Electrical and Electronics Engineering Department, Antalya International (Bilim) University, Turkey. His current research interests include wireless physical and MAC layers security, orthogonal frequency-division multiplexing multiple-input multiple-output systems, advanced waveforms design, multi-dimensional modulation techniques, and orthogonal/non-orthogonal multiple access schemes for future wireless systems. He served as a TPC Member for several international conferences. He is a Regular Reviewer for various IEEE, Elsevier, and Springer.