APPLICATION OF THE ARCH and MS-ARCH MODELS **TO THE IBEX-35** RETURN SERIES

Abstract

In the analysis of financial time series, the Autoregressive models with Conditional Heteroskedasticity (ARCH) and their generalization, the GARCH models, have been widely used, demonstrating their good qualities for modeling the volatilities typical of this type of series. As an alternative, Switching Markov models have emerged that allow the inclusion of random phenomena as possible structural changes in the mean or variance process. This paper aims to demonstrate the best suitability of these regime-switching processes for modeling the conditional variance of the IBEX-35 Index returns.

Keywords: Conditional volatility, GARCH models, Switching Markov models, Regime, Stock index, Conditional heteroskedasticity.

1. INTRODUCTION

Financial time series have different characteristics from the time series usually studied. The conditional variance is changeable over time and not observable, which requires a treatment by models that explain this particularity.

In the analysis of financial time series, Autoregressive models with Conditional Heteroscedasticity (ARCH) and its generalization (GARCH) have demonstrated their greater suitability for the forecast of conditional variance than methodologies that start from the hypothesis of constant variance over time. However, (Klaassen, 2002) warns that these models do not behave efficiently in periods of financial stress, due to the high persistence exerted by large innovations on forecasts. [1](Lamoureux and Lastrapes, 1990) state that this fact is due to the poor specification of volatility models, that is, when the process of conditional variance undergoes structural changes that are not taken into account in the description of volatility, the persistence of conditional variance is greater.

(Hamilton and Susmel, 1994) demonstrate that the prediction of volatility in a GARCH model (1,1) described by a first-order difference equation does not conform to the regularities observed in markets, as past innovations exert greater weight on volatility forecasts as the exponential decay parameter approaches one.

The model proposed by (Cai, 1994) and later by (Hamilton and Susmel, 1994) of states with conditional heteroscedasticity, Markov Model of Switching-Regime ARCH incorporate possible structural changes experienced by the conditional variance of financial assets as a completely random phenomenon, described by an unobservable state variable. These models assume that the functional form of conditional volatility may be different between states or regimes and that it can be described depending on the state of the process (high or low volatility) since many economic series seem to follow stationary processes, but by different tranches (or periods). These sections can be explained by a [2] random, discrete, unobservable state variable explained by a Markov chain.

This study is intended to compare the different treatment that ARCH and GARCH models without change of states and the Markov Model of Switching-Regime ARCH with change of state make on the conditional volatility of the returns of the IBEX-35 Index, and to establish if the latter conform to the stylized facts observed in the financial markets, that is, if the Spanish stock market presents regime changes defined as states of high (or low) volatility, structural changes in the process of medium or variance and the persistence of large innovations.

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[1] Cambios that can be generated in the structure of financial markets or the creation of new markets.

[2] A state is a situation of high or low volatility experienced by financial markets over a certain period of time.

2. AUTOREGRESSIVE MODEL WITH ARCH CONDITIONAL HETEROSCEDASTICITY

Classical linear processes with AR and MA do not take into account in the modeling of financial time series the conditional heteroscedasticity (non-constant conditional variability throughout the series) presented by these series. Engel proposed in the eighties the models with autoregressive conditional heteroscedasticity ARCH (for its acronym in English) for the analysis of conditional volatility that most financial time series present. Over time, a multitude of extensions have emerged such as the GARCH, EGARCH, MS ARCH models, etc. that better adjust the data.

2.1. Preliminary considerations

- Financial time series are not stationary and have some kind of trend. To avoid increasing volatility, the first difference of the logarithms of returns is taken:

$$\mathrm{R_t} = \log \frac{\mathrm{X_t}}{\mathrm{X_{t-1}}}$$

- The R_t series has a leptocurtic distribution.
- R_t is white noise[3]. The autocorrelation function of the sample $\rho(l),\ l \neq 0$ is not significantly different from 0. However, it is not noise iid. Que there is no temporal correlation in a linear model does not mean that it cannot exist non-linearly[4]
- Volatility clusterig. The volatility of these series usually forms groups. Therefore, there is a positive correlation between the squares of the yields being, therefore, heteroscedastically conditioned.[5]

3. AUTOREGRESSIVE MODEL WITH GENERALIZED CONDITIONAL HETEROSCEDASTICITY (GARCH)

(Baillie and Bollerslev, 1992) they introduced a model in whose structure the conditional variance depends, in addition to the square of the delayed residues *q* periods, as in the ARCH(q) model, on the delayed conditional variances *p* periods. This model is known as conditionally heteroscedastic GARCH self-regulatory generalized.

3.1. Model Specification

Let be a stochastic process, being a discrete set of indices, and... parameter vectors to model the mean and variance respectively; the vector of explanatory variables observed in t, . In the model, y is the information available up to time t. The model is given by:$\{y_t\}\ t\ \in\ \mathrm{T}\mathrm{T}\beta' = (\beta_0, \beta_1, \dots, \beta_k)\omega' = (\alpha_0, \alpha_1 \alpha_q, \gamma_1, \dots, \gamma_p)x_t = (1, x_{t1}, \dots, x_{tk})z_t = (1, \varepsilon_{t-1}^2, \dots, \varepsilon_{t-q}^2, h_{t-1}, \dots, h_{t-p})\varepsilon_t =\ y_t \beta \psi_{t-1}$

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3 Stock indices are usually random walks, so their rates follow white noise processes.

4 $Cov(R_1, R_{i+l}) = 0$However $Cov\left(R_i^2, R_{i+l}^2\right)\ y\ Cov(|R_i|, |R_{i+l}|)$ they don't have to be equal to zero, since the data does not have to be independent.

5 The variability of yields depends on their recent changes.

$$y_t|\psi_{t-1} \sim \mathrm{N}(\mu_t, \sigma_t^2)$$
$$\mu_t = x_t\beta$$
$$\sigma_t^2 = z_t\omega = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}^2$$
$$\varepsilon_t = y_t - x_t\beta$$

4. MODELOS MARKOV SWITCHING

The previous study of the models without regime change has been necessary to establish the comparison between this type of models and those that, if they present a change of regime, all in order to find the most appropriate model for the returns of the IBEX-35.

The Markov Switching models with regime change proposed by (Hamilton, 1989) were initially used for the study of non-linear economic and financial time series with structural changes. Currently they are used in many disciplines such as medicine or meteorology. These models describe time series by an unobservable and finite random variable, defined as the realization of a stochastic process that only takes discrete values, represented by a first-order Markov chain.

4.1. Literature review

Many economic or financial time series present changes in the evolution of their media process, which are due to exogenous or endogenous interventions; similarly, they exhibit nonlinear dependency structures. Therefore, its treatment in many applications requires using tools different from the traditional ARMA or ARIMA models.

Due to the above, during the last two decades numerous instruments have been developed to analyze non-linear time series, among them the Threshold Autoregressive (TAR) models of (Tong and Lim, 1980), the Smooth Transition Autoregressive (STAR) models , neural networks, nonlinear space-state models. Also, the Markov Switching Models or models with Markov regime changes proposed by (Hamilton, 1994) stand out.

(Marcucci, 2005) evaluates forecast performance and Value at Risk (VaR) calculation using the GARCH(1.1), EGARCH(1.1) and MS-GARCH(1.1) models for the series of returns of the US *Standard and Poor's* 100 index in a period between 1988 and 2003 on a daily basis. The author finds the MS-GARCH(1,1) model superior in different prediction horizons, not being so for the calculation of the VaR where the model does not present advantages with respect to other methodologies.

To adjust the *DAX* stock market index of the German stock market (Wilfling, 2011) test the MS-GARCH models for a period between 2000 and 2009 contrasting different adjustments for conditional variance in the regimes. The author concludes that the specification of an APARCH model (from the family of asymmetric volatility models) with two possible volatility states is the one that best fits the data series.

In order to explain the structure of dependence on energy prices in the Scandinavian countries (interconnected by a common electrical structure, but with different energy generating processes) (Haldrup and Nielsen, 2006) they affirm that, although the price remains relatively the same in each country, in circumstances of production restrictions the price may be different and subject to regime changes in line with the productive capacities. The authors propose a model where persistence in each state may be different.

(Abounoori et al., 2016) compare different GARCH specifications and MS-GARCH models to make forecasts on the *tehran Stock Exchange* index with horizons of 22 days. The models are also evaluated for the calculation of the VaR. The authors presented 27 models ranging from GARCH(1,1) without regime change, to an MS-AR-GARCH(1,1), the latter model being the one that presented a better behavior to forecast the performance of the index.

Finally, many of the applications of the different methodologies that allow to explain the structure of dependence of the returns of the financial assets are motivated by the verification of the hypothesis of the efficiency of the markets defined by (Fama, 1965; Samuelson, 1965)

the hypothesis of market efficiency as defined (Samuelson, 1965) postulates that the information available to all market agents is reflected in the price of assets, which implies that prices are impossible to predict or equivalently the forecast of the series of returns conditioned on the entire set of y_{t+1}available information is zeroF_t. One of the topics studied in relation to the market efficiency hypothesis is anomalies in financial assets. (Lo, 2008) defines this phenomenon as "a regular pattern in the returns of an asset that is reliable, widely known, and inexplicable." This definition refers to the seasonal patterns present in the processes generating the return data of financial assets. Research on these patterns they conducted (Lakonishok and Smidt, 1988) for the *S&P500* index revealed the following results:

- **Monthly regularities**: On average, asset returns differ between the first and second half of each month, being higher for the first half of the month, except for the last half of December. Likewise, there is a differential effect in the month of January associated with better returns of small companies in the United States. Finally, the authors find no evidence in favor of seasonal patterns for the rest of the months when the data are analyzed for each month as a whole and not by subsamples between months.
- **Weekend effect**: Evidence is found in favor that there are significant differences between the returns of the different days of the week, particularly the returns turn out to be negative for Mondays or first business day of the week, while for each Friday or last business day of the week, they are positive.
- **Return of holidays**: According to the authors, the returns of the working days before a holiday are higher than those associated with the Fridays in which the market normally operates. In contrast, the business day after the holiday is greater than the return of a usual Monday.
- **Month change effect**: Its results indicate that asset returns are higher between the five business days that are around the change of month.

4.2. Markov Switching Model Autoregressive of the first order, with two possible regimes.

Defined by the equation:

$$y_t = \theta_{s_t} + \phi_{s_t} y_{t-1} + \sigma_{s_t}\epsilon_t, s_t = \{1,2\}$$

Where and is the realization of an observable stochastic process with correlation structure between past and present realizations, $\epsilon_t \backsim i.i.d.\mathrm{N}(0,1)$ and $\alpha_t = \sigma_{s_t}\epsilon_t$, represents the innovations of each period. The random variable s_t marks the current regime of the process in each period t, which allows to have two dependency structures . The probability law for s_t is explained by a Markov chain of the first order of the type:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Where P is the transition matrix showing the probability that the process will pass to a state $s_t = i$ since in t-1 it was in $s_{t-1} = j$, with i,j = 1,2.

Thus, each column of the matrix P runs through the entire probabilistic space of the variable state $\{s_t\}$ conditioned to the realization in the immediately preceding period, $\{s_{t-1}\}$; therefore, the sum of each element $P_{i1} + P_{i2} = 1$, for i = 1, 2. In addition, for the process $\{s_t\}$ other important characteristics are assumed such as constant transition probabilities in time, ergodic (stationary) Markov chains, without absorbent states ($0 < P_{ij} < 1$) and non-periodic (Hamilton, 1994).

(Hamilton, 1994) points out that the Scheme of Markov chains can be used to describe the process followed by the unobservable variable $\{s_t\}$ because it generates significant forecasts. This type of model includes the possibility of short-term and atypical events occurring in very long periods of time such as events characterized by low returns and high volatility that would refer to a probable change of regime. the st process is described by a first-order autoregressive vector [VAR(1)]. To do this, define the vector:

$$\xi_t = \begin{bmatrix} (1,0)' \; si \; s_t = 1 \\ (0,1)' si \; s_t = 2 \end{bmatrix}$$

Then, knowing that the transition probabilities represent a conditional expected value, the expected value of the process in the period s_tt + 1 can be obtained, as follows:

$$E|\xi_{t+1}|s_t = i| = \begin{bmatrix} P_{i1} \\ P_{i2} \end{bmatrix}$$

$E|\xi_{t+1}|\xi_t| = P\xi_t$Therefore, and since the process s_t follows a first-order Markov chain$E|\xi_{t+1}|\xi_t|\xi_t, \xi_{t-1}, \xi_{t-2} \ldots.]$, then the result is obtained:

$$\xi_{t+1} = P\xi_t + v_{t+1}$$

Where v_{t+1} is a sequence of random variables in which, at a given time,

the conditional hope of the next value of the sequence, given all the above values, is equal to the present value. It is, therefore, a sequence in differences with zero mean.

If we call the forecasts as F-steps ahead of the process $\{y_t\}$, we describe ξ_{t+F} and their expected value up to the available information in t, using the following equations:

$$\xi_{t+F} = v_{t+F} + Pv_{t+F-1} + P^2 v_{t+F-2} + \cdots + P^{F-1} v_{t+1} + P^F \xi_t$$

$$E[\xi_{t+F}|\xi_t, \xi_{t-1}, \ldots] = P^F \xi_t$$

The above equations are fundamental in the process of estimating the parameters and in the inference of the realizations of the latent process $\{s_t\}$. In addition, they are essential for the construction of the F forecasts of the process.$\{y_t\}$

4.3. Estimate

Two are the approaches mainly used to obtain the estimators of the parameter and the variable $\{s_t\}$. The first of these is the Maximum Likelihood Method that through the hope-maximization algorithm or EM algorithm, allows to obtain consistent and asymptotically normal estimates of the parameters and reconstruct the process $\{s_t\}$ by means of an iterative procedure (Douc et al., 2004). The second approach proposes Bayesian estimation methods from Monte Carlo simulations that allow to obtain estimates of the parameters and the state variable together.

4.3.1 Maximum Likelihood Method

(Augustyniak, 2014) development this method to obtain estimators of the parameters in specifications with components of moving averages in the mean process or GARCH parameters in the equation that describes the dynamics of conditional variance by implementing the EM algorithm, this allows to replace the realizations of the state variable $s = \{s_0, s_1, \dots, s_T\}$ by their expected values conditioned to the data that we will denote as $\mathrm{P}(s_t = j | y_t, \dots, y_0; \Theta)$. The process will be repeated until the likelihood function is maximized (under some convergence criterion). The procedure is outlined in the following steps:

- Propose a set of initial values for each of the parameter vector components $\Theta^{(0)}$
- Starting from an initial value for the beginning of the iterative schema$\mathrm{P}\left(s_t = j \middle| y_t, \dots, y_0; \Theta^{(0)}\right)$
- Iteratively calculate the expected values of the state variable given the data and the initial values of the parameters $s_t(y_0, y_1, \dots, y_{t-1})\Theta^{(0)}$ for t=1, t=2,...,t=T, or know the equation:

$$\mathrm{P}\left(s_t = j \middle| y_t, \dots, y_0; \Theta^{(0)}\right) = \sum_{i=1}^{2} \mathrm{P}_{ji}\, \mathrm{P}\left(s_t = j \middle| y_t, \dots, y_0; \Theta^{(0)}\right), i, j = 1,2$$

- Once you have the above values it is necessary to obtain the values of the density function of the data unconditioned to the state variable, for t=1, t=2,...,t=T
- With all the elements it is necessary to evaluate the likelihood function and obtain a new set of values for the parameters$\Theta^{(1)}$
- Starting from $\Theta^{(0)}$the entire described process is repeated until the maximum of the likelihood function is obtained or some convergence criterion is met.

4.3.2 Bayesian estimation approach

This methodology proposed by (Bauwens et al., 2010) in order to obtain the parameter vector estimators Θ and the reconstruction of the process $\{s_t\}$ through the use of Bayesian methods. The authors develop this technique from three blocks.

The first of these is composed of the entire sequence of the state variable$s = \{s_0, s_1, \dots, s_T\}$. The second is composed of the transition probabilities of the process $\{s_t\}$, $\mathrm{P} = \{\mathrm{P}_{11}, \mathrm{P}_{12}\mathrm{P}_{21}\mathrm{P}_{22}\}$. The third is made up of the vector $\Theta = (\theta_1, \theta_2, \phi_1, \phi_2, \omega_1, \omega_2, \alpha_1, \alpha_2, \beta_1, \beta_2)$.

The general scheme consists of assigning a set of initial values to $\mathrm{P}^{(0)}$ and $\Theta^{(0)}$, proceeding to take a random sample of the probability density functions of the state variable for each realization of the sequence $s^{(1)} = \left\{s_1^{(1)}, s_2^{(1)}, \dots, s_T^{(1)}\right\}$ conditioned to the initial values of $\mathrm{P}^{(0)}$ and $\Theta^{(0)}$. Next, a random sample taken from the probability density function is obtained P thus obtaining a set of possible values $\mathrm{P}^{(1)}$ conditioned to the sequence $\{s^{(1)}\}$ y $\Theta^{(0)}$.

Finally, a random sample of the probability density function of the Θ vector conditioned to $s^{(1)}$ and $\mathrm{P}^{(1)}$, achieving a possible realization of the parameter vector denoted by $\Theta^{(1)}$. The above process is performed about 50,000 times. To make the inference on the parameters and the state variable, the last 30,000 possible realizations of the different blocks are left (Bauwens et al., 2010); represented as follows:

$$\{s^{(30.000)}, s^{(30.001)}, \dots, s^{(50.000)}\}$$

$$\{\mathrm{P}^{(30.000)}, \mathrm{P}^{(30.001)}, \dots, \mathrm{P}^{(50.000)}\}$$

$$\{\Theta^{(30.000)}, \Theta^{(30.001)}, \dots, \Theta^{(50.000)}\}$$

4.4. Applying the model to the series

For the modelling of the series of returns of the IBEX-35 index, the Markov Switching Autoregressive model will be implemented following the study I carry out (Hamilton, 1989) for the growth rate of the gross domestic product of the United States.

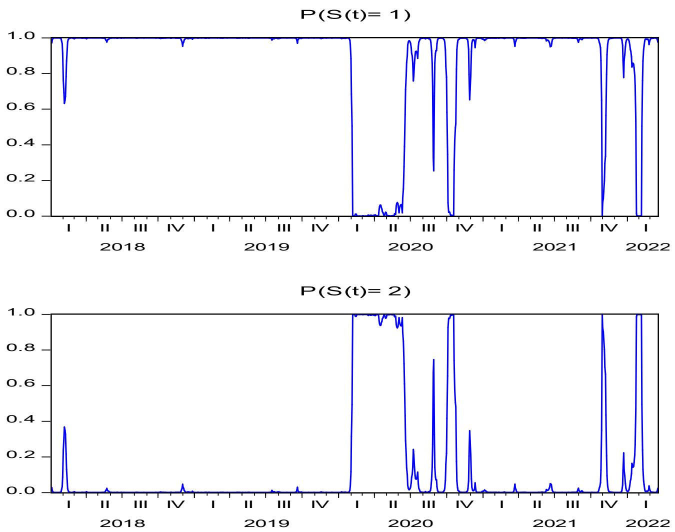

Figure 4.1 Markov Switching AR(1). Smoothed probability regimes

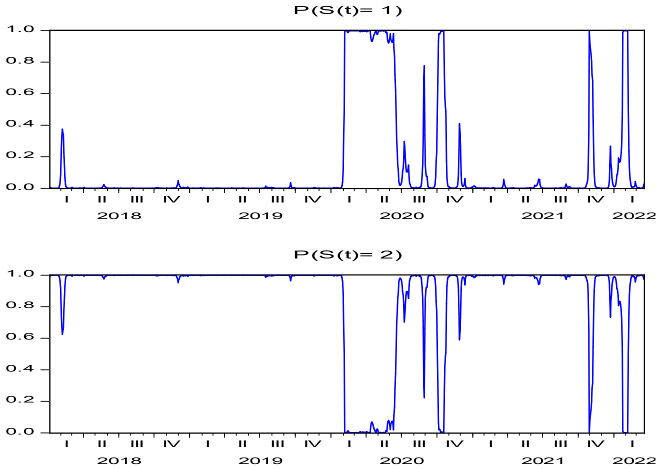

Figure 4.2 Markov Switching AR(2). Smoothed probability regimes

The Markov Switching AR(2) specification, although having statistically significant coefficients, does not provide new information, in addition the graphical analysis confirms the inversion of the regimes.

4.4.1 Markov Switching AR(4)

The result of the estimate is shown in Table

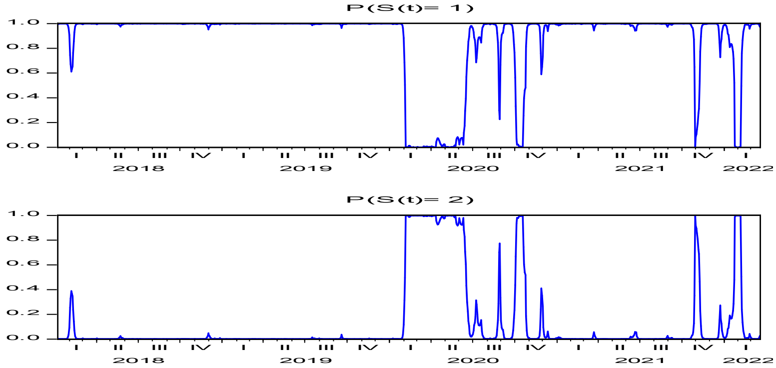

Figure 4.3 Markov Switching AR(4). Smoothed probability regimes

4.5. Model selection

For the selection of the model, the information criteria with their respective values are shown in Table 4.1

Information criteria	MS AR(1)	MS AR(4)
Akaike	-6. 193243	-6.189677
Black	-6. 161382	-6.144062
Hanna-Quinn	-6. 181189	-6. 172417

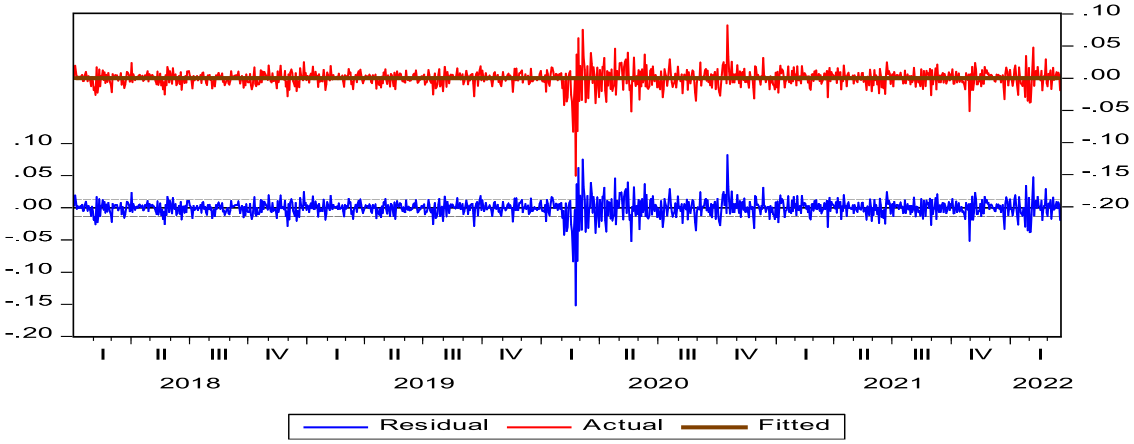

Figure 4.4 Adjustment of the model without regime change to the returns of the IBEX-35

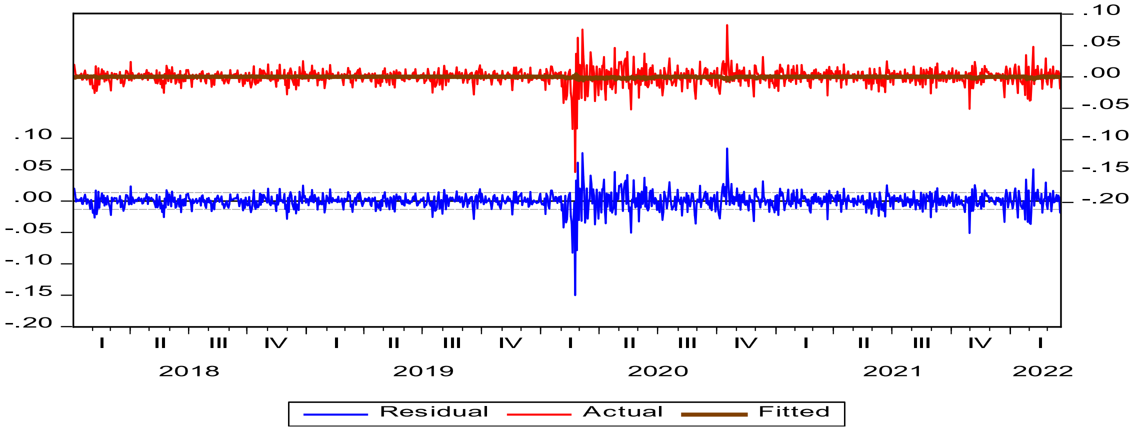

Figure 4.5 Adjustment of the model with regime change to the returns of the IBEX-35

4.6. Comparison between models with change and without change of regime

For the selection of the model, the information criteria with their respective values are shown in Table 4.2

Information criteria	ARCH(4)	MS AR(1)
Akaike	-6.157673	-6.193243
Black	-6.130383	-6.161382
Hanna-Quinn	-6.147346	-6.181189

Table 4.2 Information criteria for models ARCH(4) and MS AR(1)

The MS AR(1) model obtains the lower information criteria and, therefore, is determined to be the best model for modelling the returns of the IBEX-35 index.

5. CONCLUSIONS AND RECOMMENDATIONS

The present study aimed to answer the question of whether the return dependency structure of the IBEX-35 index could be adequately explained by Markov Switching models with conditional heteroscedasticity, compared to models that do not take into account regime changes. For this, the selected sample covers the beginning of the covid-19 pandemic, this fact being considered as a border between regimes. The methodology has consisted of a process of modeling, validation and the subsequent use of the models found for the realization of forecasts. We decided to implement a proposal in the search for possible models with bottom-up Way regime change, which consists of proposing very simple models and reviewing performance measures in each adjustment. A detailed analysis of the available information was carried out , both quantitative (summary statistics and statistical contrasts), and qualitative (specific events that could mark the history of the process) and models that do not incorporate regime change were proposed. in order to establish comparisons with Markov Switching models, ensuring that they pass the different validation tests. It was determined that the process generating the data is an MS-AR(1) with two possible regimes, whose transition matrix of probabilities indicates a higher probability of remaining in the origin regime (0.99). The corresponding expected durations for each regimen are 130 days and 19, respectively. The superiority of this model for the modeling of financial series is contrasted by comparing the information criteria Schwarz (SBIC), Hannan-Quinn (HQC) and Akaike (AIC) that is established between the models without change of regime and their subsequent comparison with the switching models.

The proposed methodology for the determination of a model with regime change has been satisfactory judging by the results obtained in the validations of the models and the comparison of these through the different information criteria.

The models of the GARCH family, in particular the GARCH(1,1) have been superior in that they have obtained the values of the lowest information criteria. It would therefore be advisable to compare these models with the Markov-Switching GARCH models, to establish the suitability of these models in the treatment of conditional volatility. The comparison of these models exceeds the objectives of this study.

Because there is no consolidated framework for the determination of models that efficiently describe the behavior of the different financial series, it is interesting to develop this type of methodology that allows to find, with a certain level of confidence, the true model that generates the data with regime change.

6. REFERENCES

Abounoori, E., Elmi, Z., y Nademi, Y. (2016). Forecasting Tehran stock exchange volatility; Markov switching GARCH approach. Physica A: Statistical Mechanics and its Applications, 445, 264-282.

Augustyniak, M. (2014). Maximum likelihood estimation of the Markov-switching GARCH model. Computational Statistics & Data Analysis, 76, 61-75.

Baek, C., Fortuna, N., y Pipiras, V. (2014). Can Markov switching model generate long memory? Economics Letters, 124(1), 117-121.

Baillie, R. T., y Bollerslev, T. (1992). Prediction in dynamic models with time-dependent conditional variances. Journal of Econometrics, 52(1-2), 91-113.

Bauwens, L., Preminger, A., y Rombouts, J. V. K. (2010). Theory and inference for a Markov switching GARCH model. The Econometrics Journal, 13(2), 218-244.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

Cai, J. (1994). A markov model of switching-regime arch. Journal of Business and Economic Statistics, 12(3), 309-316.

Carrasco, M., Hu, L., y Ploberger, W. (2014). Optimal Test for Markov Switching Parameters. Econometrica, 82(2), 765-784.

Cheung, Y. W., y Lai, K. S. (1997). Bandwidth Selection, Prewhitening, and the Power of the Phillips-Perron Test. Econometric Theory, 13(5), 679-691.

Cho, J. S., y White, H. (2007). Testing for Regime Switching. Econometrica, 75(6), 1671-1720.

Dark, J. (2015). Futures hedging with Markov switching vector error correction FIEGARCH and FIAPARCH. Journal of Banking & Finance, 61, S269-S285.

Davidson, J. (2004). Forecasting Markov-switching dynamic, conditionally heteroscedastic processes. Statistics & Probability Letters, 68(2), 137-147.

Di Sanzo, S. (2007). Testing for linearity in Markov switching models: a bootstrap approach. Statistical Methods and Applications 2007 18:2, 18(2), 153-168.

Dickey, D. A., y Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. Journal of the American Statistical Association, 74(366), 427.

Diebold, F. X., y Inoue, A. (2001). Long memory and regime switching. Journal of Econometrics, 105(1), 131-159.

Douc, R., Moulines, É., y Rydén, T. (2004). Asymptotic properties of the maximum likelihood estimator in autoregressive models with Markov regime., 32(5), 2254-2304.

Enders, W. (1995). Applied econometric time series. En Applied Econometric Time Series. Wiley.

Engle, R. F. (1982). A general approach to lagrange multiplier model diagnostics. Journal of Econometrics, 20(1), 83-104.

Engle, R. F., y Patton, A. J. (2007). What good is a volatility model? Forecasting Volatility in the Financial Markets, 47-63.

Engle, R. F., y Yoo, B. S. (1987). Forecasting and testing in co-integrated systems. Journal of Econometrics, 35(1), 143-159.

Engle, R. F., Ito, T., Lin, W.-L., Campbell, J., Granger, C., Kane, A., Rothchild, M., y Sims, C. (1988). Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market.

Fama, E. F. (1965). Portfolio Analysis in a Stable Paretian Market. 11(3), 404-419.

Garman, M. B., y Klass, M. J. (1980). On the Estimation of Security Price Volatilities from Historical Data. The Journal of Business, 53(1), 67.

Geweke, J., y Porter-Hudak, S. (1983). The estimation and application of long memory time series models. Journal of Time Series Analysis, 4(4), 221-238.

Giraitis, L., Kokoszka, P., Leipus, R., y Teyssière, G. (2003). Rescaled variance and related tests for long memory in volatility and levels. Journal of Econometrics, 112(2), 265-294.

Glosten, L. R., Jagannathan, R., Runkle, D. E., Breen, W., Hansen, L., Hess, P., Hsieh, D., Judson, R., Kocher-Lakota, N., Mcdonald, R., Nelson, D., y Siegel, D. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. The Journal of Finance, 48(5), 1779-1801.

Haldrup, N., y Nielsen, M. Ø. (2006). A regime switching long memory model for electricity prices. Journal of Econometrics, 135(1-2), 349-376.

Hamilton, J. D. (1994). Chapter 50 State-space models. Handbook of Econometrics, 4, 3039-3080.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica, 57(2), 357.

Hamilton, J. D., y Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. Journal of Econometrics, 64(1-2), 307-333.

Harris, D., McCabe, B., y Leybourne, S. (2008). Testing For Long Memory. Econometric Theory, 24(1), 143-175.

Hasbrouck, J. (1999). The Dynamics of Discrete Bid and Ask Quotes. The Journal of Finance, 54(6), 2109-2142.

Hobijn, B., Franses, P. H., y Ooms, M. (2004). Generalizations of the KPSS-test for stationarity. Statistica Neerlandica, 58(4), 483-502.

Hwang, S., y Satchell, S. (s. f.). Modelling Emerging Market Risk Premia Using Higher Moments.

Klaassen, F. (2002). Improving GARCH volatility forecasts with regime switching GARCH. Advances in Markov-Switching Models, 223-254.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., y Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? Journal of Econometrics, 54(1-3), 159-178.

Lakonishok, J., y Smidt, S. (1988). Are Seasonal Anomalies Real? A Ninety-Year Perspective. The Review of Financial Studies, 1(4), 403-425.

Lamoureux, C. G., y Lastrapes, W. D. (1990). Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects. The Journal of Finance, 45(1), 221-229.

Lee, D., y Schmidt, P. (1996). On the power of the KPSS test of stationarity against fractionally integrated alternatives. Journal of Econometrics, 73(1), 285-302.

Lee, S., y Masuda, H. (2010). Jarque–Bera normality test for the driving Lévy process of a discretely observed univariate SDE. Statistical Inference for Stochastic Processes 2010 13:2, 13(2), 147-161

Ljung, G. M., y Box, G. E. P. (1978). On a measure of lack of fit in time series models. Biometrika, 65(2), 297-303.

Lo, A. W. (2008). Efficient Markets Hypothesis. The New Palgrave Dictionary of Economics, 1-17.

Lo, A. W., Durlauf, S. N., y Blume, L. E. (s. f.). E Efficient Markets Hypothesis.

Mackinnon, J. G., Mackinnon, J. G., y Mackinnon, J. G. (1991). Critical values for cointegration tests. EDS.), Long-Run economic relationships: readings in cointegration.

Marcucci, J. (2005). Forecasting stock market volatility with regime switching GARCH models. Studies in Nonlinear Dynamics and Econometrics, 9(4),

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59(2), 347.

Nyblom, J. (1989). Testing for the constancy of parameters over time. Journal of the American Statistical Association, 84(405), 223-230.

Parkinson, M. (1980). The Extreme Value Method for Estimating the Variance of the Rate of Return. Source: The Journal of Business, 53(1), 61-65.

Perron, P. (1989). The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis. Econometrica, 57(6), 1361.

Phillips, P. C. B. (1987). Towards a unified asymptotic theory for autoregression. Biometrika, 74(3), 535-547.

Phillips, P. C. B., y Perron, P. (1988). Testing for a unit root in time series regression. Biometrika, 75(2), 335-346.

Polwitoon, S., y Tawatnuntachai, O. (2008). Emerging market bond funds: A comprehensive analysis. Financial Review, 43(1), 51-84.

Ray, B. K., y Tsay, R. S. (2002). Bayesian methods for change-point detection in long-range dependent processes. Journal of Time Series Analysis, 23(6), 687-705.

Roll, R. (1984). A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. The Journal of Finance, 39(4), 1127-1139.

Said, S. E., y Dickey, D. A. (1984). Testing for unit roots in autoregressive-moving average models of unknown order. Biometrika, 71(3), 599-607.

Samuelson, P. A. (1965). Using Full Duality to Show that Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand. Econometrica, 33(4), 781.

Tong, H., y Lim, K. S. (1980). Threshold Autoregression, Limit Cycles and Cyclical Data. Journal of the Royal Statistical Society: Series B (Methodological), 42(3), 245-268.

Wilfling, B. (2011). Markov-switching GARCH models in finance: a unifying framework with an application to the German stock market.