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Type of the Paper: Conference Paper

# Measuring vertical tyre stiffness of bicycle tyres

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Name of Editor: Firstname Lastname

Submitted: dd/mm/yyyy

Accepted: dd/mm/yyyy

Published: dd/mm/yyyy

Citation:

## Abstract:

This contribution presents an analysis of the vertical tyre stiffness of 20" bicycle tyres as usually mounted on bicycle carriers for the transport of children. The current research contributes to the science on bicycle comfort with the focus on the next generation cyclists. Two different methods to measure vertical or radial tyre stiffness of bicycle tyres are presented – a dynamic approach on a dynamic press and a static approach. Parameters modified are tyre inflation pressure and vertical load in the static experiment. In the dynamic experiment additionally dynamic load and frequency are varied. The dynamic experiments are performed on two different tyres. The same tyres are also used for the static experiments and completed with a third tyre, which is a clincher version of the narrow foldable tyre. The tyres are made for 406mm rim diameter as usually for bicycle carriers since the comfort of children in bicycle transportation is the larger scope behind the experiments.

The main findings are as follows:

- The stiffness of the tyres is in a range of 31 N/mm to 147 N/mm. It must be considered that values below 50 N/mm are related to extremely low inflation pressure that probably do not work reliably because the rim will puncture the tube.
- Tyre inflation pressure is the main factor that controls the vertical stiffness.
- Type of tyre (balloon vs. narrow tyre) hardly affects the stiffness.
- The dynamic stiffness at 1 Hz is slightly higher than the static stiffness.
- With increasing excitation frequency the stiffness increases, however, this effect is non-linear and varies between 3.7% at high pressure in the narrow tyre and up to 20% at low pressure in the balloon tyre.
- Similarly, there is a trend to higher stiffness with increasing vertical load in a magnitude of 20% increase.

**Keywords:** bicycle tyre, vertical tyre stiffness, radial tyre stiffness, tyre suspension, dynamic stiffness, static stiffness

## Introduction

Studying bicycle carrier comfort is an important research topic for at least three reasons. First, it is closely related to sustainable mobility, as the problem of low-emission and (sub-)urban daily mobility is often planned to be solved through an increased use of bicycles. Second, bicycle as mode of transportation is more and more in focus even for simulation tools supporting planning of infrastructure. So, there is need for empirical data for the simulation of bicycles as means of transport. Third, child transportation in bicycle carriers and cargo bikes makes a significant fraction of child transportation by bike. However, there is lack of data about the vibration level and the experienced comfort.

In cycling there is a development to wider tyres, even in racing cycling. This contributes to a decrease of rolling resistance, however, an increase of comfort is always assumed but not yet shown systematically. Regarding bicycle carriers wider tyres (balloon tyres) are often used in combination with lower inflation pressure to increase comfort. Anyhow, in previous experiments lower tyre inflation pressure did not automatically correspond to decreased level of acceleration i. e. increased comfort Rothhämel (2023).

However, ride comfort or the absence of discomfort are important motivation factors for utility cycling. Hagemester and Schmidt (2003) found that good surface quality and the absence of curbs has highest priority for cyclists in their route choice, which is strongly connected to ride comfort. One solution (A) is to improve the surface quality within bicycle infrastructure, another solution (B) is to make bicycles more robust against bad surface quality. In reality the choice will not be A or B only. In addition, both of them are embedded in compromises against other factors as also Hagemester describes.

The tyre and its properties, specifically stiffness and eigenfrequencies, are important steps to improve the understanding of ride comfort on bicycles and in cycle carriers.

## Earlier research

Several investigations were done for tyres on agriculture tractors. Lines and Murphy (1991) found that the radial stiffness increased linearly with the inflation pressure superposed by an offset of the carcass stiffness. However, they found rim diameter, tyre section width, tyre age, and inflation pressure to be the significant parameters affecting tyre stiffness. Therefore, the suggested to replace the conceptual model describing tyre stiffness by means of an equivalent spring and damper by means of an empirical formula including these factors.

Brassart and Wright (1993) measured vertical stiffness of agriculture tractor tyres on a test rig in a static and a dynamic way. Next to the already known linear correlation between tyre inflation pressure and static stiffness, they found that the dynamic behaviour differs from the static. In spite of the fact that the excitation was sinusoidal, this result is in contrast to the findings by Zegelaar (1998).

In his thesis Zegelaar (1998) made comprehensive measurements of automotive tyres. He found that the vertical stiffness of rolling tyres differed from the stiffness of non-rolling tyres when excited randomly but hardly when excited sinusoidally.

In context with a multibody simulation of bicycles Waechter et al. (2002) measured tyre stiffness on two different bikes. The tyres were not specified in detail, however, the stiffness measured was  $200 \text{ kN/m}$  and  $134 \text{ kN/m}$  respectively.

Lepine et al. (2016) measured in-situ two different wheel sets (including different tyres) on system level and could distinguish the two wheel-sets.

Maier et al. (2018) investigated mainly longitudinal characteristics of a bicycle tyre but also measured vertical stiffness.

Doria et al. (2019) performed a modal analysis of complete utility bicycles and identified bounce and pitch modes in the range of 10-15 Hz based on tyre deformation. To verify some of their findings, they measured the static vertical tyre stiffness in an isolated setup. They found a certain non-linear behaviour at loads around 100 N but a quite linear behaviour at higher loads that are of more importance, however, no tyre modes were identified. In a later experiment Doria et al. (2021) tested a numerical method to predict the comfort of a city bike and identified frequency response functions of the complete bicycle including the tyres. Most of the peaks were in a frequency range larger than 20 Hz.

In contrast, Rothhämel (2023); Rothhämel and Liu (2023) found when investigating the system comfort of children transported in a bicycle carrier, next to a dependency of speed and inflation pressure, peaks at 3.3 Hz and higher. Anyhow, cycle carriers have a different architecture that might influence how the tyre characteristics affects the results.

## Scope

The scope of this publication is to contribute knowledge around the stiffness and eigenfrequencies of bicycle tyres and to improve the understanding of the parameters such as type of tyre, inflation pressure, static and dynamic vertical load. In addition, two different measurement methods are applied, once a static method, once a dynamic method, with the goal to verify each other or to understand differences in the tyre behaviour.

## Method

### Tyre setup

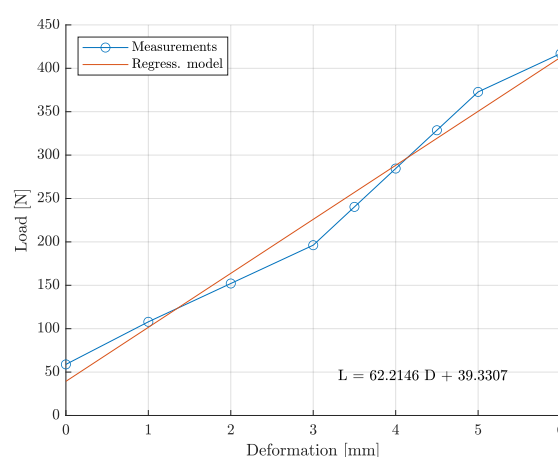
The wheel used for this investigation consisted of a 20" rim with 36 spokes. The tyres were a narrow one of the type Schwalbe Kojak in the ETRTO<sup>1</sup> dimension 35-406 once in a foldable, once in a non-foldable version and a balloon tyre of the type Schwalbe Big Apple in the dimension 60-406 non-foldable. The tyres were mounted with corresponding butyl rubber tubes. The tyre inflation pressure was varied between the tyre individual maximum and a fraction of the minimum specified on the tyre. The pressure was measured with a commercially available indicator that could unfortunately not be calibrated. However, the tyre pressure was measured before and after each series of experiments, which gave at least consistent results.

### Static vertical tyre stiffness

The static vertical tyre stiffness ( $k_{V,stat}$ ) was measured by means of scales and a metering rule. The tyres were mounted on a cycle carrier that was placed on three scales. The carrier was levelled by means of a lifting jack under the hitch, see Fig. 1. The tyre inflation pressure was set and load was added bit by bit. For each combination of tyre inflation pressure and load the vertical deflection as well as the vertical force were measured. At six different levels of inflation pressure and nine sets of load measurements were taken (Fig. 2). For each level of inflation pressure ( $p_{infl}$ ) the corresponding vertical tyre stiffness was calculated by means of linear regression analysis. The coefficient describing the inclination was set as stiffness at this inflation pressure.



**Figure 1.** Cycle carrier on scales for static vertical tyre stiffness measurements.



**Figure 2.** Load (L) over deformation (D) for the balloon tyre (60-406) at 150 kPa.

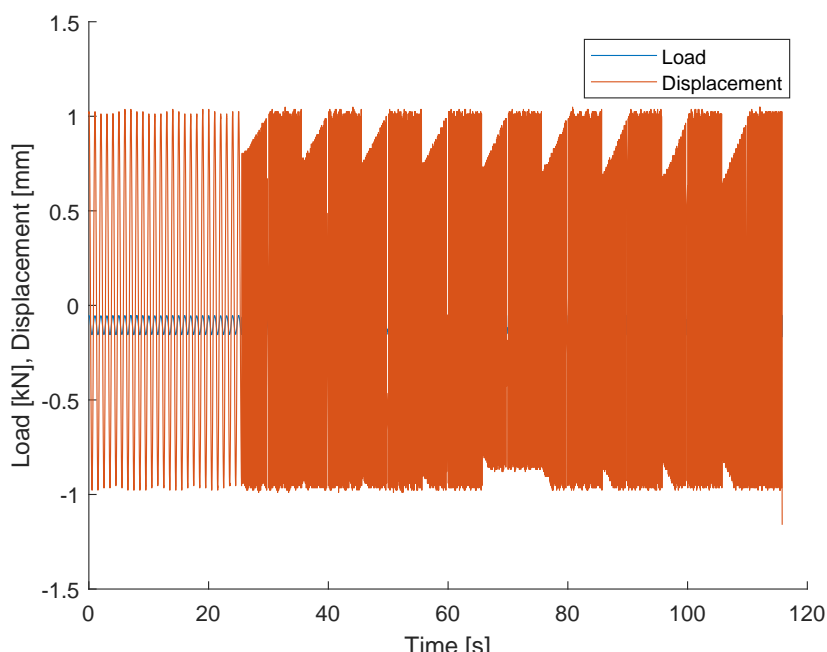
<sup>1</sup>European Tyre and Rim Technical Organisation: [www.etrto.org](http://www.etrto.org)

## Dynamic vertical tyre stiffness

The dynamic vertical tyre stiffness ( $k_{V,dyn}$ ) was measured by means of a dynamic press as shown in Fig. 3.



**Figure 3.** Bicycle wheel in a dynamic press for vertical stiffness measurements (narrow tyre: 35-406).



**Figure 4.** Load and displacement over time for the balloon tyre (60-406) at 400 kPa.

Adaptors were designed and manufactured to guarantee a planar surface. In addition, the surface was covered with abrasive paper with grain size P80 as usual in tyre testing to simulate a fine asphalt surface. The tyre was pressed in-between a piston and an overarm. This means that the tyre suspension took place two times, once on the upper contact patch and once on the lower contact patch. However, the force acting vertically is the same. This was considered in the evaluation by dividing the displacement by two.

The machine was controlled by displacement. Therefore, for each setting a static pre-test was performed to define the necessary displacement. Deviations of the force were accepted. A deviation of the results was not expected because corresponding force and displacement were measured simultaneously and evaluated accordingly.

For each setting the tyre pressure was measured first. Then, the tyre was loaded at a frequency of 1 Hz to avoid settlements during the measurements. After that, the pre-test was done to map the controller. This was done manually, increasing the force by means of the controller, reading the resulting displacement. The actual test was performed three times always beginning with a 1 Hz sine over a time span of 25 s followed by 10 s each at higher frequencies (first line Table 1). Because of preliminary results the focus was moved to a higher resolution of frequencies around 7 Hz. Therefore, a second set of frequencies was generated (see second line in Table 1). The higher the frequency was controlled, the higher was the deviation of the machine. The controller could obviously not steer the hydraulics quick enough.

**Table 1.** Frequencies for testing dynamic tyre stiffness.

1 <sup>st</sup> setting	1	1.7	2.2	2.8	3.6	4.7	6.0	7.8	10	Hz
2 <sup>nd</sup> setting	1	6	6.5	7	7.5	8	8.5	9	9.5	10 Hz



## Results

### Static tyre stiffness results

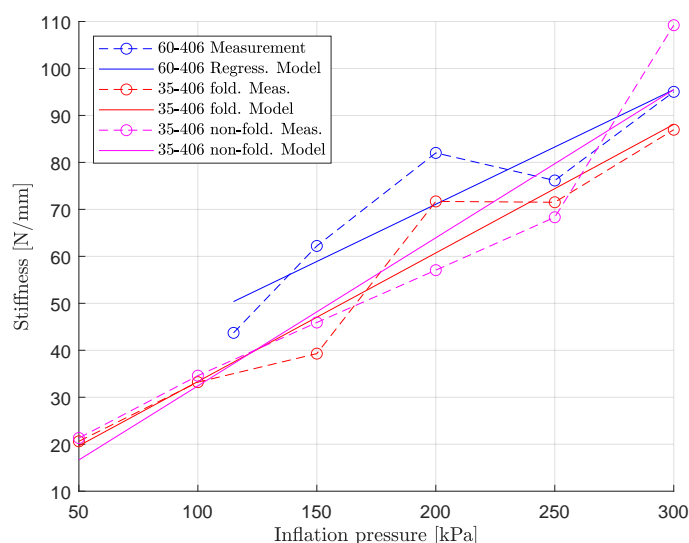
The force and deflection characteristics as shown in Fig. 2 were evaluated and an according stiffness was identified for each tyre inflation pressure. In a regression analysis this characteristics was summarised in coincidence with equation (1). The corresponding coefficients are shown in Table 2. In addition, the stiffness is visualised in Fig. 5.

$$k_{V,stat}(p_{infl}) = b_1 \cdot p_{infl} + b_2 \quad (1)$$

**Table 2.** Coefficient for Equation (1) including the 95% confidence intervals (CI).

Tyre	$b_1 [\frac{N}{mm}/kPa] \pm CI$	$b_2 [N/mm] \pm CI$
35-406 foldable	$0.274 \pm 0.092$	$5.983 \pm 17.86$
35-406 non-foldable	$0.315 \pm 0.132$	$0.891 \pm 25.77$
60-406 non-foldable	$0.244 \pm 0.185$	$22.34 \pm 39.61$

According to Lines and Murphy (1991)  $b_2$  in Equation (1) and Table 2 can be seen as the tyre's own carcass stiffness of  $k_{V,stat,0}$  independent of inflation pressure. In contrast  $b_1$  is the gradient of the graph that indicates how the stiffness increases over an inflation pressure range from 50 to 300 kPa. The  $b_2$  coefficients indicate a higher tyre own stiffness for the wider tyre, which corresponds with the subjective impression when handling the tyres. However, the  $b_2$  confidence intervals (not shown in the figure) for all tyres include zero. Negative values are technically not meaningful. Anyhow, the tyres cannot be distinguished significantly with regard to their stiffness based on these measurements. This is valid for both of the coefficients.

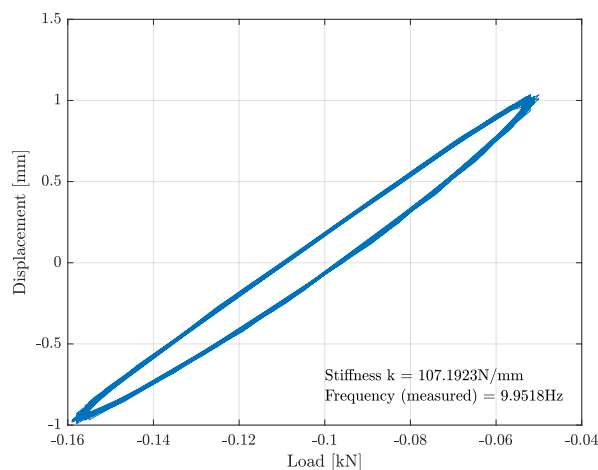


**Figure 5.** Vertical tyre stiffness over tyre inflation pressure for three types of tyres and the corresponding models based on regression analysis. The confidence intervals (not shown here) overlap widely.

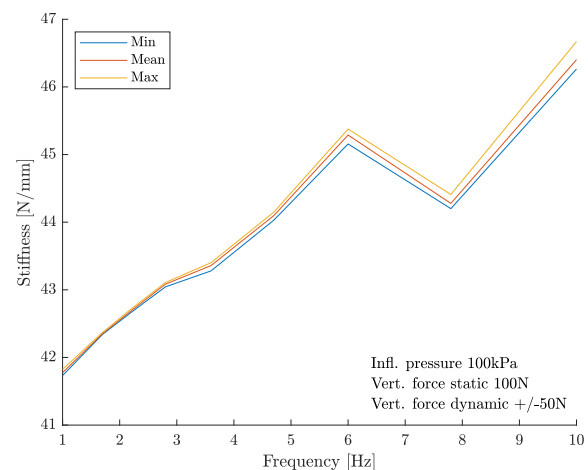
### Dynamic tyre stiffness results

The measurements as shown in Fig. 4 were evaluated in a script where the inclination of the mid 50% of the hysteresis loop were approximated by means of a linear function. An interim result is shown in Fig. 6. The frequency shows an error of 5% in comparison to the specified value.

The results of several measurements were averaged for each setting consisting of inflation pressure, load (pre-load and dynamic load variation) and frequency. An example plot is given in Fig. 7.



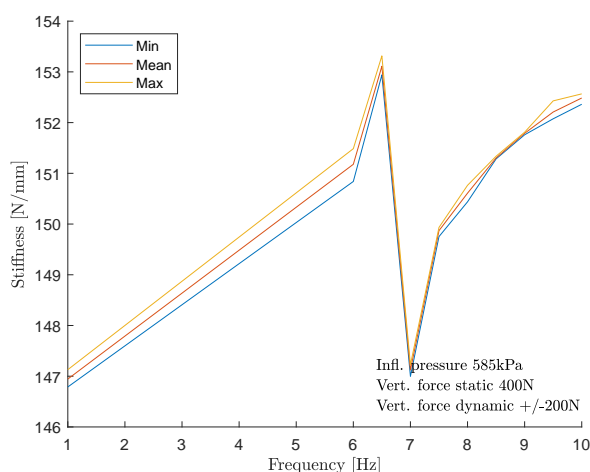
**Figure 6.** Single measurement of displacement over vertical load at 10 Hz and 400 kPa inflation pressure (tyre: 60-406).



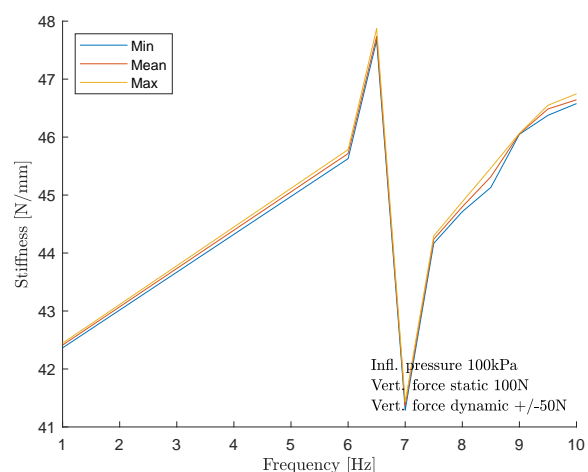
**Figure 7.** Dynamic vertical stiffness measurements at 100 kPa inflation pressure and  $100 \pm 50$  Nm vertical load (tyre: 35-406).

The results show a general increase of stiffness over up to 12% from 1 Hz to 10 Hz. For very low tyre inflation pressure in the balloon tyre the increase can reach 20%. Independent of tyre pressure and load a clear local minimum was available at 7.8 Hz, which could be understood and during the experiment experienced as some kind of resonance. Therefore, further investigations focussed more on this frequency range using another setting of frequencies for testing (see second line in Table 1). A test for one setting with higher frequencies (10 – 30 Hz) did not show large changes and specifically not a certain trend. This triggered the decision to focus on the frequency range below 10 Hz only.

Fig. 8 and 9 show the results with focus on 6 – 10 Hz. The downwards peak at 7 Hz is much more distinct than in Fig. 7. In addition, it can be seen that there is a certain deviation e. g. 2% at 1 Hz in Fig. 8, which shows room for improvements within repeatability. When investigating Fig. 8 it must be noted that the inflation pressure is not equal to the desired 650 kPa. During the second measurement campaign a problem with the air pressure system occurred in the lab, which lead to this deviation.



**Figure 8.** Dynamic vertical stiffness measurements with focus on 6 – 10 Hz at 585 kPa inflation pressure and  $400 \pm 200$  Nm vertical load (tyre: 35-406).

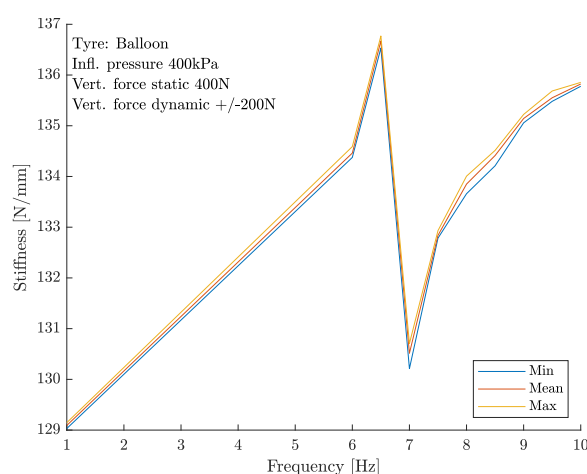


**Figure 9.** Dynamic vertical stiffness measurements with focus on 6 – 10 Hz at 100 kPa inflation pressure and  $100 \pm 50$  Nm vertical load (tyre: 35-406).

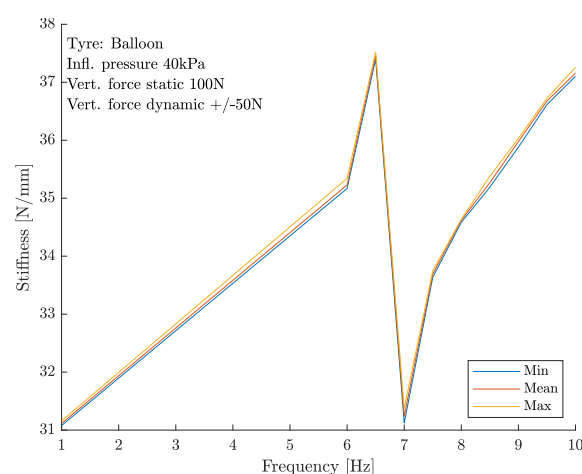
Next to the experiments on the narrow tyre (see above), a balloon tyre was tested with about three times the air volume (see Table 3), where the tyre volume was approximated by means of the geometric figure of a torus. The results are shown in Fig. 10 and 11. Even in these experiments the downwards peak at 7 Hz is clearly visible. In a later test of the machine without the bicycle wheel, it could be shown that the machine has an eigenfrequency at 6.8 Hz! This means that a superposition of the tyre properties and the test machine must be assumed where the effects of the machine probably will overbalance.

**Table 3.** Comparison of narrow and balloon tyre.

Tyre	Volume
35-406 (non-)foldable	1.333 l
60-406 non-foldable	4.139 l



**Figure 10.** Dynamic vertical stiffness measurements with focus on 6 – 10 Hz at 400 kPa inflation pressure and  $400 \pm 200$  Nm vertical load (tyre: 60-406).



**Figure 11.** Dynamic vertical stiffness measurements with focus on 6 – 10 Hz at 40 kPa inflation pressure and  $100 \pm 50$  Nm vertical load (tyre: 60-406).

## Discussion

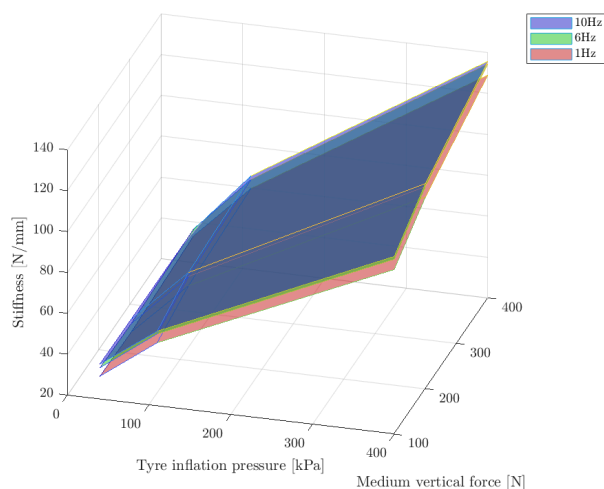
Two different methods to measure vertical or radial tyre stiffness of bicycle tyres were presented – a dynamic approach on a hydropuls machine and a static approach. Parameters modified were tyre inflation pressure and vertical load in the static experiment. In the dynamic experiment additionally dynamic load and frequency were varied. The dynamic experiments were performed on two different tyres. The same tyres were used for the static experiments and completed with a third tyre, which was a clincher version of the narrow foldable tyre. The tyres were made for 406 mm rim diameter as usually for bicycle carriers since the comfort of children in bicycle transportation was the larger scope behind the experiments.

The stiffness of the tyres was in a range of  $31 \text{ N/mm}$  to  $147 \text{ N/mm}$  in the dynamic measurements and in the range of  $20 \text{ N/mm}$  to  $110 \text{ N/mm}$  in the static measurements. The differences depend on different tyre pressure ranges. It must be considered that values below  $50 \text{ N/mm}$  are related to inflation pressure and tyre combinations that probably do not work reliably because the rim will puncture the tube. This coincides with the values known from literature as e.g. by Waechter et al. (2002) who identified  $200 \text{ N/mm}$  on a recumbent bike and  $134 \text{ N/mm}$  on a special upride bike both with 20" wheels as in the present study, without any more specifications about type of tyre and inflation pressure. The tyre stiffness measured by Maier et al. (2018) is with  $173 \text{ N/mm}$  at 475 kPa inflation pressure significant higher. Two major differences to the present experiments are that Maier et al. tested a 28" tyre (42-622) and that they tested with higher load (up to 1500N). In their plot the stiffness at lower vertical load is obviously lower and closer to the here presented findings.

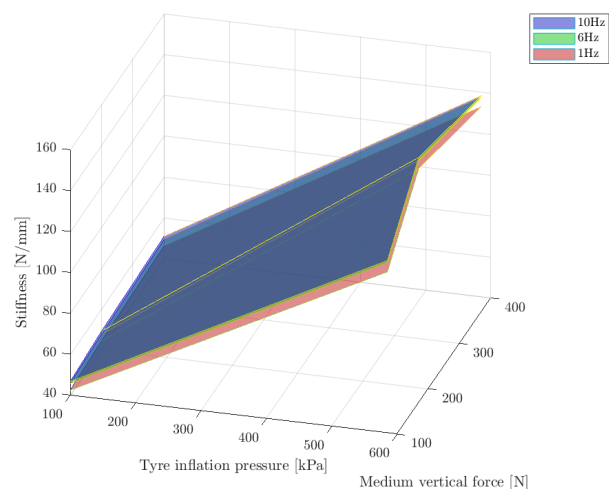
Considering the magnitude of the stiffness, the characteristics of the different tyres tested in this experiments, cannot be distinguished. The parameter with significant influence seems to be the inflation pressure. This is also valid for the different measurement

methods when considering the dynamic measurement at 1 Hz as nearly static. In contrast, with increasing excitation frequency the stiffness increases, however, this effect is non-linear and varies between 3.7% at high pressure in the narrow tyre and up to 20% at low pressure in the balloon tyre. Similarly, there is a trend to higher stiffness with increasing vertical load in a magnitude of 20% increase. This is in contrast to the findings of Doria et al. (2019) who found for a not in detail specified tyre with 300 kPa inflation pressure at low vertical load (< 100 N) a stiffness of 38 N/mm and at high vertical load (> 200 N) a stiffness of 91 N/mm, which corresponds to an increase of 140%.

All results of the dynamic tyre stiffness experiment are summarised presented in Fig. 12 for the balloon tyre and in Fig. 13 for the narrow tyre.



**Figure 12.** Dynamic vertical stiffness measurements over inflation pressure and vertical load at different excitation frequencies (tyre: 60-406).



**Figure 13.** Dynamic vertical stiffness measurements over inflation pressure and vertical load at different excitation frequencies (tyre: 35-406).

Eigenfrequencies of the tyre were not investigated any longer after the superposed effect of the dynamic press and its dominating eigenfrequency around 6.8 Hz.

## Conclusion

In this study, we investigated the dynamic stiffness characteristics of various types of tires under different conditions of excitation frequency and vertical load.

First, the most important factor for vertical tyre stiffness is the tyre inflation pressure. Second, dynamic measurements at 1 Hz cannot be interpreted as static, specifically because of the uncertainty of the static measurements. In addition, in the dynamic measurements there is a small but clear increase in vertical tyre stiffness with increasing excitation frequency. Third, over increasing vertical load no clear stiffness characteristic could be identified.

This suggests that tire stiffness is slightly dependent on the excitation frequency and tire type, emphasizing the need for careful consideration when designing tires for specific applications.

The advantage of the presented dynamic method is that it can be utilised in a machine that is available in many labs. The disadvantage is that the path of the vertical load does not correspond with the path when the wheel is mounted at a bicycle or bicycle carrier. A further study is suggested where the presented measuring method will be compared to a fork mounted wheel. In addition, a comparison to larger wheels 559 mm or 622 mm is recommended to clarify a generalisation of the results.

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